Hardy-Weinberg Equilibrium

Hardy-Weinberg Equilibrium, also referred to as the Hardy-Weinberg principle, is used to compare allele frequencies in a given population over a period of time. A population of alleles must meet five rules in order to be considered "in equilibrium":

1) No gene mutations may occur and therefore allele changes do not occur.
2) There must be no migration of individuals either into or out of the population.
3) Random mating must occur, meaning individuals mate by chance.
4) No genetic drift, a chance change in allele frequency, may occur.
5) No natural selection, a change in allele frequency due to environment, may occur.

Hardy-Weinberg Equilibrium never occurs in nature because there is always at least one rule being violated. Hardy-Weinberg Equilibrium is an ideal state that provides a baseline against which scientists measure gene evolution in a given population. The Hardy-Weinberg equations can be used for any population; the population does not need to be in equilibrium.

There are two equations necessary to solve a Hardy-Weinberg Equilibrium question:

$$p + q = 1$$
$$p^2 + 2pq + q^2 = 1$$

p is the frequency of the dominant allele.
q is the frequency of the recessive allele.
p^2 is the frequency of individuals with the homozygous dominant genotype.
$2pq$ is the frequency of individuals with the heterozygous genotype.
q^2 is the frequency of individuals with the homozygous recessive genotype.

Example 1a:
A population of cats can be either black or white; the black allele (B) has complete dominance over the white allele (b). Given a population of 1,000 cats, 840 black and 160 white, determine the allele frequency, the frequency of individuals per genotype, and number of individuals per genotype.

To solve this problem, solve for all the preceding variables ($p, q, p^2, 2pq, and\ q^2$).

Step 1: Find the frequency of white cats, the homozygous recessive genotype, as they have only one genotype, bb. Black cats can have either the genotype Bb or the genotype BB, and therefore, the frequency cannot be directly determined.

$$Frequency\ of\ individuals = \frac{Individuals}{Total\ Population}$$

$$\frac{Individuals}{Total\ Population} = \frac{160}{1{,}000} = 0.16$$

Frequency of white cats = 0.16; therefore, $q^2 = 0.16$

Step 2: Find q by taking the square root of q^2.

$$\sqrt{(q^2)} = \sqrt{(0.16)}$$
$$q = 0.4$$

Step 3: Use the first Hardy-Weinberg equation ($p + q = 1$) to solve for p.

$$p + q = 1$$
$$p = 1 - q$$
$$p = 1 - (0.4)$$
$$p = 0.6$$

Now that the allele frequencies in the population are known, solve for the remaining frequency of individuals by using $p^2 + 2pq + q^2 = 1$.

Step 4: Square p to find p^2.

$$p = 0.6$$
$$p^2 = (0.6)^2$$
$$p^2 = 0.36$$

Step 5: Multiply $2 \times p \times q$ to get $2pq$.

$$2pq = 2(0.6)(0.4)$$
$$2pq = 0.48$$

Therefore:
The frequency of the dominant alleles: $p = 0.6$
The frequency of the recessive alleles: $q = 0.4$
The frequency of individuals with the dominant genotype: $p^2 = 0.36$
The frequency of individuals with the heterozygous genotype: $2pq = 0.48$
The frequency of individuals with the recessive genotype: $q^2 = 0.16$

Remember: Frequencies can be checked by substituting the values above back into the Hardy-Weinberg equations.

$$0.6 + 0.4 = 1$$
$$0.36 + 0.48 + 0.16 = 1$$

Step 6: Multiply the frequency of individuals ($p^2, 2pq, and\ q^2$) by the total population to get the number of individuals with that given genotype.

$p^2 \times total\ population = 0.36 \times 1{,}000 = 360$ black cats, BB genotype.
$2pq \times total\ population = 0.48 \times 1{,}000 = 480$ black cats, Bb genotype.
$q^2 \times total\ population = 0.16 \times 1{,}000 = 160$ white cats, bb genotype.

Comparing Generations
To know if a population is in Hardy-Weinberg Equilibrium scientists have to observe *at least two* generations. If the allele frequencies are the same for both generations then the population is in Hardy-Weinberg Equilibrium.

Example 1b:
Recall: the previous generation had allele frequencies of $p = 0.6$ and $q = 0.4$.
The next generation of cats has a total population of 800 cats, 672 black and 128 white. Is the population in Hardy-Weinberg Equilibrium?

Step 1: Solve for q^2.

$$\frac{Individuals\ with\ the\ Recessive\ Genotype}{Total\ Population} = \frac{128}{800} = 0.16$$

Step 2: Use q^2 to solve for q. There is no need to solve the entire equation, because if q has changed, then p has also changed. If q remains the same, then p will remain the same.

$$q^2 = 0.16$$
$$\sqrt{(q^2)} = \sqrt{(0.16)}$$
$$q = 0.4$$

Because the recessive allele frequency (q) has remained the same, the population is in a state of Hardy-Weinberg Equilibrium.

Example 2a:
The beak color of finches has a complete dominance relationship where black beaks are dominant over yellow beaks. There are 210 individuals with the genotype DD, 245 individuals with the genotype Dd and 45 individuals with the genotype dd.
Find: the frequency of the dominant and recessive alleles and the frequency of individuals with dominant, heterozygous, and recessive traits.

Step 1: Add up all the individuals to calculate the total population.

$$210 + 245 + 45 = 500$$

Step 2: Find q^2.

$$\frac{Individuals\ with\ Recessive\ Genotype}{Total\ Population} = \frac{45}{500} = 0.09$$

Step 3: Take the square root of q^2 to find q.

$$q^2 = 0.09$$
$$\sqrt{(q^2)} = \sqrt{(0.09)}$$
$$q = 0.3$$

Step 4: Use the first Hardy-Weinberg equation ($p + q = 1$) to solve for p.

$$p + q = 1$$
$$p = 1 - q$$
$$p = 1 - (0.3)$$
$$p = 0.7$$

Now that the allele frequencies in the population are known, solve for the frequency of all individuals by using $p^2 + 2pq + q^2 = 1$.

Step 5: Square p to find p^2.

$$p = 0.7$$
$$p^2 = (0.7)^2$$
$$p^2 = 0.49$$

Step 6: Multiply $2 \times p \times q$ to get $2pq$.

$$2pq = 2 \times 0.7 \times 0.3$$
$$2pq = 0.42$$

Therefore:
The frequency of the dominant alleles: $p = 0.7$
The frequency of the recessive alleles: $q = 0.3$
The frequency of individuals with the dominant genotype: $p^2 = 0.49$
The frequency of individuals with the heterozygous genotype: $2pq = 0.42$
The frequency of individuals with the recessive genotype: $q^2 = 0.09$

Example 2b:
The next generation of finches has a population of 400. There are 336 with black beaks and 64 with yellow beaks. Is this population in Hardy-Weinberg Equilibrium?

Step 1: Solve for q^2**.**

$$\frac{\textit{Individuals with the Recessive Genotype}}{\textit{Total Population}} = \frac{64}{400} = 0.16$$

Step 2: Take the square root of q^2 to find q.

$$q^2 = 0.16$$
$$\sqrt{(q^2)} = \sqrt{(0.16)}$$
$$q = 0.4$$

Because the recessive allele frequency (q) has changed, the population is **NOT** in a state of Hardy-Weinberg Equilibrium.

Practice Problems

1. Scale coloration of lizards has a complete dominance relationship where green scales are dominant over blue scales. There are 1,024 individuals with the genotype GG, 512 individuals with the genotype Gg, and 64 individuals with the genotype gg.
   Find: the frequency of the dominant and recessive alleles and the frequency of individuals with dominant, heterozygous, and recessive genotype.

2. The next generation of lizards has 1092 individuals with green scales and 108 individuals with blue scales. Is the population in Hardy-Weinberg Equilibrium? Solve for p and q.

3. Rabbit's ears can be either short or floppy, where short ears are dominant over floppy ears. There are 653 individuals in a population. 104 rabbits have floppy ears and 549 have short ears.
   Find: the frequency of the dominant and recessive alleles and the frequency of individuals with dominant, heterozygous, and recessive genotypes.

4. The next generation of rabbits has 560 individuals with short ears and 840 individuals with floppy ears. Is the population in Hardy-Weinberg Equilibrium? Solve for p and q.

5. Petal coloration of pea plants has a complete dominance relationship where purple petals are dominant over white petals. There are 276 plants, 273 have purple petals.
   Find: the frequency of the dominant and recessive alleles and the frequency of individuals with the dominant, heterozygous, and recessive genotype.

6. The next generation of pea plants has 552 plants, 546 have purple petals. Is the population in Hardy-Weinberg Equilibrium? Solve for p and q.

Solutions

1. $p = 0.8$
   $q = 0.2$
   $p^2 = 0.64$
   $2pq = 0.32$
   $q^2 = 0.04$

2. $p = 0.7$
   $q = 0.3$
   No, the population is not in a state of Hardy-Weinberg Equilibrium because the allele frequencies are not the same as the preceding generation.

3. $p = 0.6$
   $q = 0.4$
   $p^2 = 0.36$
   $2pq = 0.48$
   $q^2 = 0.16$

4. $p = 0.23$
   $q = 0.77$
   No, the population is not in a state of Hardy-Weinberg Equilibrium.

5. $p = 0.9$
   $q = 0.1$
   $p^2 = 0.81$
   $2pq = 0.18$
   $q^2 = 0.01$

6. $p = 0.9$
   $q = 0.1$
   Yes, the population is in a state of Hardy-Weinberg Equilibrium.

References

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